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The records of that university show that Professor Royce gave three lectures as follows:

I
Thursday, November 3
"The Orderly Arrangement of facts and ideas: 'Series' and 'Levels'"

II
Friday, November 4
"Transformations and Their Laws"

III
Saturday, November 5
"The World and the Will"

J. Harry Cotton

September 4th, 1953
Richmond Lectures on "Some Fundamental Conceptions of Science".

Lecture I.

The Orderly Arrangement of Facts and Ideas: Series and Levels.

I propose, in the following lectures, to point out the philosophical bearing of a few ideas with which, in a sense, you are all more or less familiar. I have called these ideas in my title: "Some Fundamental Conceptions of Science". In this lecture I shall try to tell you what these ideas are, and why they are...
fundamental, and why I have ventured to trouble you with the
mention and the examination of them.

Since there is a good deal to say about
these ideas, you will forgive me if I
spend little time upon introductory
speeches, but plunge at once into the
heart of my undertaking. One
word only, as I begin, regarding
my personal attitude towards this
Audience and this undertaking.

You have very kindly in-
vited me to this institution, and to
this community, to address you upon
some topics connected with philosophy. I
delight at this opportunity to become more
closely acquainted with you, and add to
further academic and personal rela-
tions between your community and ours.

My lectures, as I fear, will contain
their titles, and perhaps too; if judged by their contents, will
be to you to deal with very dry and remote
topics. I shall every think this misappren-
tence. It seems to be the most prominent
one in your minds; but as a fact I
want, so far as I can, in the brief
time, to show you that precisely these
problems, dry and remote though
they may seem, have a very real
relation to the practical lives of all of us. If I fail in this effort to apply philosophy to life, it will not be from good will. Nor will it be because philosophy itself, if rightly interpreted, is uninspiring. In any case, I beg you to be patient of especially the earlier portions of my discussion seem to you to be essentially technical, and, hopefully, distant from practical interest. Before I am done with the whole task, I intend to reach less arid topics, and more obviously vital considerations. In the present lecture, it shall be essentially concerned with mind and body matters; and some of these will seem to be decidedly

undecided.
forbidding. If you can bear with me while I speak of these matters, what will follow in these lectures will be, although more interesting.

One of the principal problems of the thought of the present time is this: How much can our natural sciences tell us about the true nature of the world that we live in? To this question you will find a great variety of answers given. If, in the first place, you ask the special students of any such science as physics or zoology what that science teaches us about
the universe, you will generally get, in substance, the answer that physics tells us about certain properties of matter and does not attempt to tell us any more about the true nature of things than physical experience itself enables us to verify; while zoology investigates animal life, and does so by the aid of observation and experiment, and tells us precisely as much about the nature of things as its own range of observation and experiment reveals, and no more. We're told the only sort of answers that we ever got to the question about what it is which the special sciences of nature reveal to us concerning the
universe, we should of course by no means lose our interest in the existence and in the progress of the natural sciences. We should prize physics for its control over natural phenomena that figures those who know it, and advance in its application of this and zoology for the knowledge of the processes of animal life which it is gathering; but we should not feel the concern that some of us feel in the true relations between science and philosophy, for in the hearing of science, religion's answer, as a fact, unities, there are some doubts of the special sciences, and many philosophers and theologians, who do not feel satisfied with this first answer. Some of these students give us a second answer to this question, an answer which you all have heard. They tell us
namely, that the lesson of science is wider and deeper than the lesson which you could learn by merely mastering the details of even a large number of special sciences. Were such a mastery possible to you.

This lesson of science is, they say, that the comprehensive, no, the definitive, knowledge of the combined work of many sciences is something decidedly definite has been made out concerning what the whole nature of things is. Science, they say, not only one science alone, but science as an organism, has taught us that the whole universe is subject to certain types of law—mechanical law (to say some such students); evolutionary law (to say others); while some (men such as the late Herbert Spencer) attempt what they
regard as an unification of science, and try to show us how some vast
process of evolution, which includes in
its sweep physical, chemical, vital, and
total mental, and social processes, draws
from the very mechanism of things,
and that the entire senseable
world is one treasure house of examples
of a single type of natural processes.
Some of these unifiers of science prefer
materialistic formulas; some of them
attempt to reconcile science and theology;
and some of them try to interpret the entire
world idea as an expression of a
mental process — that is, of a process
similar to what goes on in our own
minds, when we think and choose. On the other hand, many of these general
students of science lay stress upon the
limitations of our powers to know the
nature of things,—limitations which,
as they say, the very study of science
emphasizes. Spencer, as you know,
drew such a lesson when, in one part
of his system, he defined, so far as that
was possible, the Unknowable, and
showed, as he held, how hopeless it is
for man to try to penetrate the mysteries
of the true nature of the universe, as
it exists apart from our experience.
Many others tell you that the very act of
successes of science, showing us as they do, that all our actual knowledge of things depends upon observation and experiment, prove that we can never yet, in our study of nature, beyond that observation and experiment verify, so that any philosophical effort to define the ultimate truth of things must necessarily fail. According to such views, the general lesson of scientific study is that you never can hope to find out whether God exists, or whether the soul is immortal, or whether the universe is simply a complex of material atoms, subject to the laws of motion, unless you can
submit all these problems to the test of observation and experiment. If the telescope or the laboratory could, by searching, find out God as something observable, then submit the hypothesis of his existence to an absolutely crucial empirical test, that the problem of God's existence would become a scientific and experimental problem, and science could deal with it. Otherwise, say such investigators, speculation is vain. God's existence remains unknowable. And a similar result follows in case of the other problems mentioned.

Thus then comes the need to get from comparative study of the results of science a genuinely unified and enlightening view of the real nature...
7 things. Some emphasize the fact that science, by virtue of the very methods which have led to its successes, teaches that since observation and experiment, which always lead to special results only, are our guides, no enlightening view of the whole nature of things can ever be hoped for. Some, like Spences, distinguish the knowable from the unknowable, and attempt to unify the one, but forbid us to enquire into the other. And thus, between the glorification of the light that science brings us, and the insistence upon the modesty that science teaches us to practice, and which it is beyond our knowledge to research, the student of philosophy often feels baffled, and to the question, what...
general

Lessons does the study of science teach us concerning the constitution of things remains, for most of our minds unanswered.

As I have pointed out, the special investigators in the various sciences are best seldom responsible for these oversimplifications. They seek to tackle the particular problems, and deal with these as they can. If we who philosophize persist in asking more general questions, that is not their fault. On the other hand, I cannot think that we are wrong in bearing in mind the pressing necessity of these more general problems about the nature of things, or in asking whether

either the methods or the results of
the special sciences have any definite
bearing upon universal problems. The
universe we have always with us. You
cannot live without believing as if you
had solved the problems of philosophy
in some very profound and important
sense. The act dutifully implies a trust
in the essential reality of the real. To
feel confidence in life implies a faith
that this world has an ideal value.

Even to undertake the study of special
science would involve a certain trust
in the reasonableness of life and of
things whose far-reaching importance
many thoughtful special investigators fail to notice. The assurance that reason rules the universe is one which involves not lightly, or painlessly, part of the past with it; or he even rationally doubts if he ever rationally doubted, in the sense in which he does so upon grounds which are essentially philosophical. And so the right to philosophize, to attack the world-problem in a serious way, is one of the essential rights of civilized men. You may not exercise the right, but you cannot lose it, except by your own fault.

I insist then, we have a right to ask what the leading science is, and in these lectures I propose to make some beginning in such
II.

New my method of inquiry in these discussions will differ from the method that some of you are accustomed to consider natural. Mr. John Fiske, throughout his career as a public teacher in this country, was accustomed to insist, as my independent follower, Herbert Spencer, upon what he believed to be the method which a comparativestudy of the results of many special sciences revealed. The doctrine of Evolution, which Fiske used so beautifully and expounded, was taught by him as such a result of the discoveries of many
sciences. And this doctrine, particularly as Fiske held, revealed in a very notable respect what the true nature of things is. Here, as you well know, was an effort to read the lesson of science by a synthesizing the generalizations of various sciences. I am not here to criticize, upon the present occasion, the usefulness of that sort of procedure. Only such a procedure is not admirer, I shall try to study the lesson of science in another way. I have just reminded you how some people can phrase their limitations of human knowledge which, as they say, the very successes of sciences bring all the more clearly to our notice.
Since observation and experiment, as well as such teachers, are our only means of finding out scientific truth, knowledge necessarily depends on observation and experiment are no rigorous guides. At the present moment I will not as yet criticize this opinion. But the general method of studying the lesson of science which this opinion exemplifies will be the method that, in another application, I shall attempt to follow. You can study the results of science, and see whether these bear upon philosophical problems. Or again, you can study the methods the ways of conceiving things, or the leading ideas of which characterize science in general, or which are notable in various sciences, and you can
Then ask how far any sort of investigation which uses these methods, which is carried on in this way, or which follows the lead of such ideas, can hope to throw any light upon the problem of the universe. This second way of studying the lessons of science is the way that I shall here follow. So what final results we shall be led, only a later lecture can show. So then, my question in this lecture will be: What lessons can we derive from a study of the general methods and fundamental conceptions upon which all scientific work is based? Whatever else our science are, they are products of human effort and activity. They stand as the embodiment of human interest. They are the products, not indeed of anybody's caprice, but certainly of the efforts of the human will. They
We not only theoretical structures. They are the outcome of practical devotion and of personally directed labors. That is one reason why they have such close relation to the industrial arts, and such vast influence over the practical activities of these days. I yield to nobody in my admiration for what is called the purely theoretical investigator—that is, the scientific worker who devotes himself to the truth 20 eagerly that he asks to hear the question asked: "Of what practical use is this research in which you are engaged?" I love to see a man 20 confident of the precariousness of mere insight that he throws worldly
success behind him, is careless of pecu
mary reward, and pen and the treasures
of his researches without stint, utterly
careless what profit others may make
out of the consequences of his devotion. Such an in
vestigator, — a Faraday, a Maxwell, a Darwin,
a Gauss, a Newton, may all the more

certainty act in the transforming of the
practical procedure, of the industrial last
of the whole spirit of a civilization, just
because he is so devoted to what he
takes to be truth for truth's sake. But
what I notice in such a man is that
he is, as a fact, engaged, despite little
theoretical remoteness, in one of the hardest
and most fruitful human
activities, — the
activity of conceiving the relation of
Things. Thinking is itself a sort of work. Theory is the building and the testing of ideal structures. And ideal structures are often harder to build than are modern ideas are often office-building. Modern concrete is much more refractory than steel beams and girders. The test that serious scientific theories have to meet are often much severer than are the tests applied to railway bridges. The engineering of a scientific plan is often much more of a task than is the spanning of a continent. In brief, the pursuit of a science is hard work; and it is work done with a human purpose, leading to a humanly interesting goal, and guided by ideals which the universe predetermines only in as far
As the universe is man's universe and in 20 jar as it permits itself to be intre-pret in man's way; so that, in one sense, the lesson of science must be linked to shurwung us, and the nature of things would look if you could leave man and man's mind out of the universe, but how man interprets his own life, and his own experience when he tries to conceive them most soberly, not rationally, and not thoroughly.

And so, my first theme in this course will be this: Science does indeed mere-ly reveal to us, not what the universe is apart from man, but how man inter-prets his own experience. This his inter-pre-tation is due to his own active efforts. If he had other ideals, if he worked other- wise, if his interests had another bent,
if his senses were awakened through other channels, if his will were the will of some angel or some demon, and not a man's will, then his science would use other conceptions, this universe would yet some other appearance, instead of nature would be defined in other fashions, his unamissed opinions would possess some other constitution. In order to read rightly then, the lesson of science, you must first see to what end and in what way the conceptions of science are built up. You must see by means of what thoughts deeds men construct this world picture, and in what way this structure of human conceptions is exemplified now by this science, and now by that.
In other words you must recognize that whatever be the facts of experience, many concepts of these facts are essential.

practical ideals which guide even the most theoretical undertakings. In sum, you must learn to estimate the importance of the subjective, of the human, of the personal factor in the life of science.

When I say this, am I more admitting that the nature of things is wholly unknowable, and that we are dealing even in the most exact science, merely with a human interpretation of things, which bears no trace of possessing any eternal value, or of being the genuine and final truth? No, I am not saying
that the nature of things is wholly unfavourable. If you wait until my closing lecture you will hear me tell why this is not my meaning. But at present I am insisting that the lesson of science is always a lesson of human life, and cannot be a revelation of what the world would be if our human nature could be left out of it, and if the world could be studied as if man were not alive at all.

Some previous speaker would admit all this I have just said; but would then attempt to illustrate it by laying their whole system against man, constraining the fact that all our ideas of the physical world depend
upon our sense organs. They would say, if we had other senses, we could have a special electromagnetic sense, in addition to our present sense of sight. We should perhaps directly feel Herzian waves, so that wireless telegraphy might appear to us as music now does. Thus, they would observe, does indeed show us how subjective and limited our present ignorance is of things.
human character, the fundamental ideas and methods upon which every science depends. In other words, I am saying, it is upon the lesson which Kant first taught us—upon the lesson that what Kant called the Categories of common thinking—the fundamental concepts upon which our sciences depend, are human categories; so that we may not imagine as the lesson of our sciences, anything about what the world would be if men the concerning thinker, the scientific worker, the organizer of this realm of empirically illustrated ideas were not an essential factor in the world as he conceived it.

This is what I want to bring to your notice. The philosophical lesson
Such considerations I have already
partly outlined; but that we must
just have, in the main, until our
final lecture. For the time being
to make you better acquainted with
among some of these fundamental
modes of conception, as the various
sciences reveal them. To this task I
now turn.

III.
The facts of experience come to
us however they happen to come.
When we are successful in the work
taught by any of our sciences, we conceive the
facts of experience in an orderly way.
Science, as you all know, is knowledge
reduced to order. The conception of order, then, is the most fundamental conception of science. If we are to reduce facts to order, we must engage in various processes of ideal construction which are usually decidedly complicated and difficult. As a result of these processes we build up certain systems of ideas to which we find our experience conforms, precisely as far as our theories of order which occur in our science. What we mean by order, our every ideal and these rational ideals sometimes call, rational ideals determine, essentially practical ideals, system of ideas offers in certain reading...
of movement in which ours will take satisfaction. In getting our ideas into order, and in adjusting these ideas to our experience, we enable ourselves to win a control over our lives such as industrial art in manifold ways impressively illustrates, while frequently, despite its frequent apparent remoteness from practical concerns, none the less constitutes one form of free and successful and satirical living. Understand what order is, therefore, and you will come to understand what science accomplishes. Understand why order is significant, and you will see why science is enlightening and powerful, and ideal. Understand that any
Science comprises but not forgetting some part of our present experience necessary knowledge. It begins to develop from science containing the assertion that all science is orderly knowledge or knowledge set in order. The all know many things; but whoever knows a science is not merely acquainted with many facts, but also possesses such principles, methods, and mastery as enable him to build together these facts as we have earlier asserted, in an orderly unity of method. Look around you during a journey, and you shall see countless beautiful features of landscape, hills, streams, valleys, cliffs, plains, headlands, mountain peaks. These are facts you know, but this is not scientific knowledge of these facts which you know. Make the journey in company with a geologist, scientifically acquainted with the country in question, and he will reduce these facts to a grand experience to some order for your delight. These cliffs are, for instance, fragments of a certain rock formation.
traceable in this or in that direction across the country, and related cliffs and hills to earlier or later formations. These cliffs and mountain peaks have specific relations to a great mountain system which extends along the shores of a whole continent. This earth's surface is full of records what the geologist, using the ideal views of antiquity of geological time, capable of regarding a geological reading. Geology involves their detecting because human experience disorder facts which, for the unscientific tourist, appear fascinating, indeed, but disconnected. What this single instance suggests, any science, so far as it is a science, illustrates. Man's attainment thermals, the further distance he goes, however, has even touched the edge any of science, knows very well the very essence of scientific mastery is the
Grouping data into such systems as the geologist
knowledge has just illustrated

is Order? The question as thus stated, has
indeed no very practical meaning. It seems
to refer to every abstract logical problem,
very abstract, yet as I must at once
point out, it is a question that meets
practical
you at every turn of life. And it also

turns has very practical bearing.

Order, people tell you, is heaven's first
Order. I shall tell you, it is in any
case the very basis of all human righteousness.
Certainly all social order, conduct and thought, are

order is not only a scientific ideal, but
it is a social, a moral, an industrial,
an aesthetic, a domestic ideal. And

wherever you meet it, its significance
is enormous. Order and civilization

go together in social affairs. The

moral life is the orderly life. We
speak of criminals as disorderly people, and define an immoral life as a disordered life. Delirium and insanity in general impress us as instances of disorderly mental life. We speak of any physical disease as a disorder. Housekeeping uses the concept of order as its most fundamental one. Without order, no domestic life. But order is not only scientific and practically essential. In many forms, it is beautiful. All the effects of art depend upon order of one sort or another. Music is an order of tones; painting, sculpture, and decorative arts involve order of lights and shades, of forms and of colors. Drama as its orderly presentation and sequence of scenes and actions. Poetry is dependent upon rhythmic order. Order
Men is everywhere where goodness, beauty, successful living, and power of spirit are to be found. The opposite order is chaos, or confusion, or destruction, or more, carelessness, or anarchy, or miscellaneous. All these names signify great to us states of things, or of ideas, which we regard with one or another form of dread or dislike, according to we are thinking of moral or aesthetic, or of scientific interests. Disorder stands for failure of one cost or another. Success, if any reasonable cost, involves order.

Thus then, the conception of Order is not merely a fundamental conception of science, but one of the most fundamental of all human ideas.
The Postman and the Apprentice, the
Lousewife and the Artist, the Philosopher
and the Moralist, all equally near
and use, in their various ways and
worlds, this idea of Order. The reason
why I especially emphasize the society
uses of the conception of Order, rather
than the other aspects of the concept
is due to my clinic to the past in any age order has
been required for us. But in fact, the conception of Order is essential
part of every civilized form of
consciousness. It is so because we need
order for the world goes on, ideas.
All this is familiar enough.

But now: What is order? What con-
stitutes order as opposed to disorder?
This question is not easy one. Only recently
have the philosophers learned here
To deal with it, Order is in some respects like what St. Augustine declared time to be. Namely, you do not ask me what time is, said Augustine, I know, but if you ask me I do not know. It is very much so with Order. If some disorderly child, whom we are trying to teach to be orderly with his playthings, clothing ducts, sports, studies, should turn upon us suddenly and ask: What is Order? could we answer him? A person might find it hard.

Let us, however, make a beginning with this question. And to this end let us turn back for the time, from the complications of the world of art, of social life, of business, and morals, and try to exemplify what Order.
means in science. Here the ideas concerned stand out a little more clearly. The instances that we shall have to study will be free from the emotional extraneous and other complications. Our view of what constitutes order will be thus rendered more accessible, and in the end more exact.

The simplest instance of a scientifically exact order of objects and ideas which I can mention to you is furnished by that most familiar of ideal objects, the series of whole numbers, one, two, three, and the rest. Little I must add
especial

your attention to the fact that, however
we may have first come by the idea
of number, the numbers names, as the
found in our minds, as the
names of a series of possible ideal
objects, — a series of objects which would
remain unaltered in character and
in properties whatever might come or
might go or might change in the
physical world. You can view the
whole numbers, if you will, as what the
mathematicians call a purely ordinal
series of ideal objects. So viewed, the numbers
for a series which has a first member
but no last member, while every
number has a next successor, and every number except one has a next predecessor, so that every number in the series has its absolutely determinate place, following everyone of a perfectly definite class of numbers, and preceding every number which is, as we usually say, greater than this number. It can be shown that by the assumption and exact statement of a very few principles of this type you can give a perfectly sufficient basis for the entire theory of the arithmetic of whole numbers.

The series of the ordinal whole numbers is an enormously important scientific conception. The
Human reason has grown greatly upon it. The earliest human science was busied with it. The higher theory of numbers forms today one of the most difficult regions of mathematical inquiry, but one where new discoveries are constantly made. Meanwhile, one may well say that all science would be impossible without the whole number and without the fractions which we define upon the basis of the whole. Yet, on the other hand, there can be no question that the numbers, as exact science now understands them, as objects whose properties are determined by pure ideas, taking account of purely ideal conceptions. Our mere experience, apart from the rational interests which lead...
to count, does not force upon us the existence of numbers, in the sense in which experience forces upon us the existence of pain or of pleasures. We want because we want to count, need to count, find rational and practical satisfaction in counting, and in its results, and gain thereby control over nature and over our own lives. But apart from the interest which a self-conscious being looks in order, and in its most fundamental instance, viz., numerical order, the existence of numbers is inconceivable.
Now the instance of the number-series is not only our first example of order; it reveals to us a fundamental property which is always present in the systems which possess order. All such systems, I affirm, are either simple series, or else complexes that are based and structured upon series. If the books in a library are in order, that is because the books are arranged in various series, which themselves are formed into series of series; so that, by knowing what alcove is a series of alcoves, what row in a series of rows, what shelf in a given series of shelves, and what position of a series of volumes on a particular shelves, you are to look for, you
can find the book. Precisely so, when the astronomer's knowledge of the fixed stars is reduced to order, the astronomer deals with a system which consists of series. There are the series of stellar magnitudes; there are statistical series containing the numbers of stars of a given magnitude which are visible in a certain portion of the heavens. There are such series as the succession of changes of brightness through which a given variable star passes. There are serial arrangements of the spectra of certain facts of various stars. Methods in astronomy, or in any other science, are set in
order by means of the countless devices of measurement, which exact science uses, are set in order by being reduced to series, and to complexes of series. Quantities form series of greater and less quantities, in which you are dealing with distances, with masses, with velocities, with pressures, or with any other measurable objects. The whole value of measurement in science may be logical explained as reducible to the advantages which result from the exactly definable series which you can get before your mind through the processes of measurement.
The simplest and best known instance of order is furnished to us by the familiar series of the whole numbers. In discovering this series the human mind made one of its first steps, and one of its most important steps towards exact science. However we came by the idea of the whole numbers,—and of that subject I have nothing to say on the present occasion—we at present conceive of them as a series of ideal objects, a series marked by certain peculiar characters. The series has a first term, namely number 1. Every term in the series is followed by a term which is called the next member of the series. As for instance, number 2 is followed by the number 3 and so on. The series has no last member. It can be proved, and has been proved in recent discussions of the logic of mathematics, that by increasing the whole number series simply in this way with the addition of a very few formal considerations, one can proceed to deduce all those properties of whole numbers which constitute the vast science of the theory of numbers, including the whole of ordinary arithmetic, so far as that arithmetic deals with whole numbers. To the whole numbers the human mind early learned to add the fractions, although in the earlier stages of mathematics the conception of what a fraction is develops somewhat slowly and in most minds remains comparatively obscure, even after considerable training in the use of numbers. The rational fractions, such as $2/3$, $9/10$, and the rest, form a very complex but very exactly ordered series of ideal objects. If any two fractions are given to you, then by the simple device of reducing them to a common denominator you can tell which of them comes earlier in the series of fractions than the other. The series of
fractions is arranged according to the so-called values of these fractions. Just as in the case of the whole numbers one can prove that the science of the theory of numbers may be developed simply by assuming the fixed order of the series of whole numbers, so one may say that the whole arithmetic of fractions depends simply upon the fact that fractions are objects which can be dealt with in certain fixed order. In fact, one can at once say that the whole of arithmetic, algebra, analysis, — that is, the whole of ordinary mathematics apart from geometry, can be developed out of the simple properties of the ordered series of whole numbers and of fractions. This very elementary instance already illustrates both the nature and the importance of the conception of order. The exactness of mathematical science depends upon the fact that the numbers are objects that stand in precise order.

But mathematical topics appear to the uninitiated extremely abstract and many of you will not be disappointed if I turn to somewhat more concrete instances of order. As I do so, I shall still call attention for the time being to cases where objects are set in order, but where the order has the same character that one may observe in the series of numbers, namely, the objects in question form a simple series of what is sometimes called a linear series, a series consisting of a single row of objects. Take a glance at any foot rule, or tape measure, or other scale used in the process of measurement. What you see at once upon the scale is an orderly arrangement of dividing points usually accompanied by numbers. Evidently the whole use of the scale depends upon the precise order of these division points and the names that are attached to them form themselves an orderly series upon which all the knowledge that can be acquired through the use of the measure depends. The thermometer that hangs
upon your wall derives again all its usefulness from the ordered series of degrees named upon the thermometer scale. The houses on one side of the street are in ordered series, numbered in a way which suggests to us in very marked fashion what the value of ordered series may be. But linear ordered series are not confined to scales or to rows of houses. The alphabet forms a single and one dimensional series of objects arranged in a single row. The various stages of any activity that we can carry out, of any list of names that we can repeat, of any orderly arrangement of the day's work; all such series form instances of the type of order that I now have in mind. Series of this type one might call artificial, since they are more or less due to our own activity as persons who are fond of getting our acts or our memories arranged in serial order, but in nature, nature gives us a large number of series of facts whose order, for the greatest importance for science as well as in many cases for art. The human voice by its well known power to run up and down the scale furnishes to us a certain natural series of tones. These we can recognize in musical instruments, and to a certain extent in the tones of nature, and upon our power to recognize this tone series depends a great deal of our skill in knowing and understanding the facts with regard to tones. But the sounds that we hear are arranged not merely according to the musical scale, they vary also in intensity, as loud sounds or faint sounds, and any given sound can vary from a greater to a less intensity by gradations which themselves constitute a simple series of the kind that we are here considering. What interests us at present is the fact not merely that such a series exists or that it has in many ways interest for our feelings; we are especially concerned with the fact that such series are of importance
to our intellect. They enable us to conceive as we say the relations of tones. The dying away of a sound indicates to us something that may have importance in telling us that the object which makes the sound is moving away from us. The increase in the intensity of a sound may indicate the approach of an object or again it may indicate the increase of some physical process which is causing the sound. In this way, you see, that a capacity to conceive the facts of nature or of our own minds in a certain serial order is the basis for a kind of scientific knowledge which if less exact than our arithmetical knowledge, still depends for its reasonableness upon similar conditions. We may here notice in passing that a part of the world of our experiences of sound is not capable of an easy arrangement in any orderly way. Or again, that many sounds are not presented to us by nature in orderly series. The sounds of the kind that we call noises vary, to be sure, in loudness. They vary also more or less in pitch, but they vary in many other ways that we cannot so easily arrange in series. Noises enable us to recognize the various things from which they come, but just because the noises cannot easily be arranged in simple series as the pitches, or the degrees of loudness, our scientific knowledge of the nature of the variety, of the classification, and of the causes of noise, either remains very much more confused and obscure than our knowledge of musical tones, or if we get such knowledge we meet with many obstacles on the way and where the obstacles are surmounted they are surmounted by means of devices which reduce again the noises to some sort of serial order.

Not only the tones, but the impressions of the sense of sight are capable of serial arrangement. These again differ in degrees
of brightness. They differ also in color. Our color sensations are of such great variety that the unaided consciousness does not easily discover the precise way in which colors are related to one another, and our popular appreciation of color remains unscientific. Our scientific knowledge of color depends in part upon the fact that by certain well known physical experiments we can form certain series of colors, namely the colors of the spectrum. Upon the basis of the discovery of this series rests very much of our further scientific acquaintance with the nature of color. In the spectrum we see our colors arranged in order, and having once seen this order we are capable of making further investigations into the relations amongst various color experiences that do not at first appear in the spectrum.

If we turn from the world of the colors to the world of the odors we get striking insteadd of the importance of order for all our scientific insight. Of the odors we have comparatively speaking very poor ideas, despite the wealth of experience that the sense of smell gives us and despite the very great interest that many of us take in the odor of perfumes, of flowers, or of foods; but the odors in the present stage of science defy our efforts to arrange them in orderly series. They are unlike the numbers, unlike the experience of musical pitch, unlike the colors of the spectrum. They are various, but they remain for our experience disordered. In another realm of our experience order again meets us, but with increasing complications and difficulties which stand in the way of our defining precise linear orders of the facts with which we deal. Our pleasures and pains, our comforts and discomforts, form to a certain extent series. The pleasure that one takes in food, in music, or in a particular conversation, may be greater or less, may
increase or decrease, and therefore one naturally comes to conceive of many of our pleasures as forming series of greater and less, and just so our pains too, our discomforts, are conceived as greater or as less. But, on the other hand, we find it impossible to arrange all pleasures in a single row. It seems impossible to decide whether a particular moral discomfort is greater or less than a particular aesthetic discomfort. If one is asked to be quite sure where in the series of displeasures a given complicated discomfort stands, he may find great difficulty in answering the question. A comparatively slight sentiment of remorse, say for what we consider a minor fault, -- is it or is it not greater than the chagrin of one who has made a social mistake, or who condemns himself for a failure to show the kind of knowledge on a given occasion he thinks he ought to show? Instances of this sort show us how hard it is to tell which of two discomforts is greater or less and to arrange all our pleasures and pains in a single series. For that very reason, however, the world of what we sometimes call values is a world especially hard to reduce to the sort of order that makes it the topic of scientific inquiry. There are many other ways of estimating values besides merely consulting our own conceptions of comfort or discomfort. The degrees of social approval or disapproval with which our acts meet tend to form series. The public disapproval with which a given kind of fault is greeted seems to be greater than that which attends some other failure of duty, and in this way we get a kind of valuation series in terms of which many people try constantly to understand what the true value of their acts may be. But here again the complexity baffles us. The verdicts of society cannot be arranged in a single series. They are
not like the numbers, they are not like the tones, they are not like the division points on a scale. As the order grows more complex our knowledge tends to be baffled. So much for an individual order. But the instances that I have just been mentioning indicate to us that in some cases where we fail to reduce things to order by arranging them in a single series, we may seek to obtain an understanding of their nature by considering the relation amongst various series of objects. As soon as I pass to this way in which we frequently endeavor to understand the natures of things, I at once come in sight of devices which are of the very greatest service in the science of thinking, what I call the correlation of series. Suppose that I am dealing with a number of objects that do not permit me to arrange them according as I say to any principle in a single series. Suppose that I can make two or more, sometimes an indefinitely great number, of relatively independent series of such objects. It still remains possible for me to understand these objects in their relations if I can connect one of these series with another in a definite way. The fact that I can do this is especially suggested to me by a familiar process upon which as a fact much of the practical and scientific value of serial order depends. If one asks the question why are we so fond of arranging objects in series, why does it help us to find things in a row at all, the instance already cited of the numbered houses on the street easily furnishes at least one answer to the question. It is convenient that houses should be arranged in a single row along the street. It is convenient that definite numbers should be attached to the various houses, and why? Because hereby as one says it is easier to find a given house. One knows how to proceed from one house to another. One can look up the address of one's friend, and know in what direction
and how far to walk to find his house. But to mention this instance
of the value of serial order is at once to remind us that in this case
the value in question depends upon the fact that there are two
distinct series of facts which in this case may be said to be correlated. My
habits make it possible for me to walk along the street in a given
direction. But this walking along the street in a given direction
is of itself a series of acts. My habits make it possible for me
to remember or to understand the order of the houses on the street.

The practical importance of the orderly arrangement of the houses de-

dpends upon the fact that I can correlate the series of my acts in
walking along the streets with the series of houses. In this way
I can answer the question as to whether I have gone further still
in a given direction to reach a given number, or whether my walk will
be a long one or a short one. Such an instance as this is one of the
simplest cases of the process of the correlation of series of which we
find instances throughout the whole range of science. In the library
the books are arranged not only on a single shelf, but on many shelves.
One finds the single book not by considering its place in a single
series, but its place in a very complex collection of series. One
may have to go to a given hall, to a given alcove, to a given row,
to a given shelf in that row, and look in a particular way for a book
of a given number.

Examples of this sort introduce us to the vast realm of complex
orders and most cases where we speak of order are instances where more
than a single series of facts are concerned. In this case a moment's
thought enables us to say that we are helped in understanding the given
order, first, by the fact that each series has its own order, and
secondly, by the fact that the many series when taken together consti-
tute a single complex order. A complex order, however, consists in
general of an interweaving or combination of various series, and
herewith we already have obtained a much more definite conception of
what order is than the conception with which we began. Arrangement
means in the first place serial arrangement. Complicated order means
combinations of various series. Let us consider a very few instances
of this type in order to supply ourselves with sufficiently varied
examples of our general concept of order.

A family tree in a treatise on genealogy constitutes an excellent
instance of a complex order. In such an order every descendant whose
name stands well down in a series of generations can be connected
with anyone of his ancestors by some perfectly definable series which
you can read off if you will examine the tree. On the other hand,
various series lead you as you examine the genealogy from one branch
of a family to another, from one person to a very distant cousin, but
meanwhile a glance serves to convince you that the tree as a whole is
formed of a certain number of what you may call fundamental series of
facts arranged in a precise way. The tree for instance shows the
descent from an ancestor who is regarded for the purposes of this
genealogy as the beginning of the family. Several children of this
ancestor are mentioned. Each one of these children has descendants
and if you follow along any particular branch of the family you get a
single series, but meanwhile, owing to the fact that there are many
children of the same generation, the new series take their beginning
in each generation and run side by side. Meanwhile when the order
is all together before you you can yourself create as it were new series
in the order that you find, and this is what you do if you ask how two
very distant cousins are, as you say, connected. Here as you see a
complex order depends upon an arrangement of various series together.

Chronology as the historians develop it is a somewhat similar complex order by means of which historians conceive the various events that interest them with reference to time. Here everything depends upon recognizing what one may call one fundamental natural series, the time series itself. The years follow one after another, and that series is a given fact behind which the historian does not attempt to go. To this series the historian correlates the various facts of any one historical sequence which interests him; the history of Egypt, the history of Greece, the history of China; the history of any state in Europe, may be thus considered and in such a history some series of notable events such as the births and deaths of kings may be seized upon for examination. This series is first correlated with the series of years. Such processes having been accomplished in case of many series, one has then a chronological system of the history of the world or some portion of the world, and this system enables the historian to make as many new correlations as he pleases. So that before long one is considering how the events in Babylonia are correlated with those in Egypt, or how the kings of England are related to those of France. Here the entire science of chronology consists of serial, and therefore orderly, arrangements of facts. Such cases as the family tree and the historical chronology may make you suppose that complex orders of the type I am now considering have a merely formal interest. They enable one, you may say, to commit facts to memory, but do they assist one in the scientific understanding of the nature of things? Well of course even in the cases just mentioned, one's comprehension of the nature of things is certainly aided by knowing that two events are contemporaneous or that certain children were
the descendants of some common ancestor.

But we may turn from such formal instances to sciences that do indeed undertake not merely to report and arrange facts but to explain their laws. Consider the kind of order the astronomical science discovers in the heavens. Astronomical science began with observing the serial orders of the positions of single planets, and of heavenly bodies, such as the sun and moon. This simple series came in the course of time to be correlated so that one could predict what phase the moon would have on a given day; or when knowledge grew greater could discover on what day an eclipse of the moon might be expected, or even perhaps an eclipse of the sun. All astronomical knowledge of our own solar system has ever since depended upon our increasing knowledge of such series and of how they are correlated. But the heavenly bodies have been from a very early stage the very type to mankind of a world of order. It is very likely if we had been unable to see the heavenly bodies we should not have obtained the conception of natural law anywhere nearly as soon as we have obtained it. Modern astronomical science is now dealing with problems far more complex and difficult than those which the solar system presents.

Now-a-days one is inquiring into the constitution of the whole stellar system, into the types of stars that are there present, into the laws that govern their appearance, their brightness, their physical constitution, their variations, their distribution, and even their whole evolution. But how does one proceed to acquire a knowledge on these vast and difficult topics? One learns to arrange facts about the stars in serial order and these serial orders grow more and more complex and more and more variously correlated as one proceeds. The stars vary in brightness. The modern astronomer deals amongst other
things with such questions as to how the stars of the various degrees of brightness, or as we commonly say, of the various magnitudes, are distributed. Are the stars of the fainter magnitudes very much more numerous than the stars of the higher magnitudes? Does the number of the stars of fainter magnitudes increase as rapidly as one would expect it to do in case the stars are somewhere nearly of the same actual size and are distributed in all parts of the heavens? Or do they cluster in certain ways and if so, according to what laws are they grouped? The physical constitution of the stars is in a great measure revealed to us by a study of their spectra. What series of spectra can spectroscopic examination of the stars reveal? Do all the stars belong to one type, or are there numerous types of physical constitution revealed by various types of spectra, and if in fact there are such various types, in what serial order can they be arranged and how shall this order be interpreted? What light does it throw if any upon the physical processes which led the stars to become what they now are? Certain stars vary in brightness, in more or less regular periods becoming more and more luminous and gradually fading away, or again, remaining steady for a long period and then for a brief period being almost extinguished. What are the various serial orders in which these phenomena occur? Again, as to these various stars, all of which are variable, do their ways of variation again form series so that some stars vary in one way, while others vary in another, and while still others lie between these two types? Upon the answer to such questions depends a knowledge with regard to the causes which make the stars vary in light and this knowledge, difficult as the problem is, is constantly growing. In a case like this you have an investigation relating to many of the most stupendous, remote, and
problematical facts of the physical world. The success of this investigation entirely depends upon the power of the astronomer to arrange observed facts in certain series. Some of these like the series of the brightnesses of the stars are so to speak forced upon him immediately by the observed fact that some stars are bright and others faint. Others of these series are so to speak much more artificial. For the different types of spectra that the stars when examined by a spectroscope show can be arranged in serial order only according to principles which the astronomer himself must more or less invent. Nevertheless whether the series are natural or artificial, upon their formation, their correlation, and the study of what results, the progress of modern astronomy depends and always will depend.

What the astronomer does in his world resembles after all what you find the chemist doing in his. A very slight study of chemistry will show you how important the conception of series is at every stage of the great science. The chemical elements themselves form a series or rather they can be arranged in a complex order which involves several related series. Upon the formation, the study, and interpretation of these series momentous problems of chemical theory depend. If one turns from such sciences in the apparently lifeless world to the world of natural history one still meets the same characteristics. Serial order is the first thing that a scientist seeks to establish. Correlations of series are the basis of all our higher knowledge of the connections of facts. Order means the existence of complex systems of interwoven series. Our scientific knowledge is a knowledge of the relations of such series. In terms of them we find facts, we know how to name them, we know
how to find them again when we need them, we learn how to measure
them, we learn how to discover the conditions which determine the
variations to which facts are subject. Every law with which we
are acquainted, if it is an exact law at all, relates to the serial
order of facts or to the connections amongst series, and this I say is
as true in the world of living forms as it is in the world of the
stars or of the chemical elements. I mentioned in the early part
of this lecture the way in which a geologist views the facts that have
to do with rock formations and mountains. A very slight acquain-
tance with geology shows you that the entire science depends upon
discovering the serial order of certain facts. The geologist has
his kind of chronology. One formation succeeds another. Two
different formations in different parts of the world are more or less
exactly contemporaneous or at any rate belong to the same general
ground geological period. Here as you see you have on one hand the series
of geological formations and on the other hand the correlations of
various series as they occur in different parts of the world. Every
explanation of geological formations depends upon the establishment
of a similar series. If we were to pass from the world of natural
history back again nearer to the practical world, to the world of
social affairs, you would again see how the establishment of serial
order among facts is the key to every exact comprehension that we
have of the connections of facts, in society as in inorganic nature.
You would also see how our inability to discover certain series which
we should like to be able to know in our study of social phenomena
limits our power to comprehend the workings of society.

But I will not weary you here with further illustration. We
set out to find out what order is. We have found that order in its
simplest form consists in the existence of certain linear series such as the series of numbers or the series of the division points on a scale. We have found that all more complex instances of order are complications made up out of series. We have already seen in a measure why serial order is so important. It is because in so far as we conceive facts in series we are able to find the single fact in its place and to understand definitely its relation to other facts. We have further seen that our knowledge of the laws of nature is very largely dependent upon our power to correlate series. That is, to find out what fact in a given series is correlated in a given way with or as I should say, is in a sort with these other facts in some other series. And so then we may say to reduce things to order is to arrange them serially and to apply similar processes to the arrangement of these series so that out of various series a complex but still orderly system results. We see that in so far as we have science we succeed in arranging our ideas of things in this way in our minds. We see that nature helps us to form these series of ours by presenting to us in a large number of cases facts that as it were come to us already arranged in series. But we see that in many other cases nature does not come in this way to our aid, but on the contrary furnishes to us facts that appear disorderly. In such cases our understanding depends upon our power to form artificial series which still more or less represent the true nature of things. We then find that all our further power to comprehend the order of things depends upon our skill in bringing into relation these various series, in correlating them, in linking one part of one series to a given part of another series according to some definite principle.

Science then has this in common with housekeeping, with the
arrangement of dinner parties, with the orderly conduct of a railway
business, with the activities of the courts of law, with all in fact
that is rational about our lives, science namely depends upon finding
or making serial order.

But the generalization which we have thus far made is after all
not very enlightening unless we are able to understand a little better
what we mean by a series itself. I spoke of the series of numbers.
I have spoken of the series of the dividing points of a scale. I
have mentioned numerous other cases of what we call series of facts,
but just as I before asked what is order, and have now answered order
is arrangement in series or in systems of series, so now I must press
the question further and with you ask what constitutes a series?
By series one plainly means a collection of objects. A single object
forms no series. Two objects can hardly be said to constitute a
series. Three objects you can arrange in series and any greater
number may constitute a series. But a series is a particular kind of
collection. It is a collection that is obviously characterized by
something very much like a law. To be able to count already tells me
something about the laws of numbers. It is very common to speak of
the law of a series. Nevertheless the term "law" is somewhat too
broad and vague a term to assist us in comprehending just what is
meant by that arrangement of objects which constitutes a series.
Because the matter is decidedly important for our subsequent under-
standing of the principles and the meaning of science, I must venture
to trouble you as I close this discussion with a few very highly
technical considerations, simple, but like most logical matter at first
sight, very dry and uninviting.
Plainly the objects which constitute a series are in some notable kind of relation to one another. To understand what I mean by series depends then upon a still more fundamental conception. It depends upon understanding what I mean by a relation. Now nothing is more familiar to the intelligent mind than the relations of things. Your family life depends upon relations. Relation of brother and sister, of parent and child, of husband and wife. All commercial and social life involve relations. To know where you are in the world is to know your space relations to other things. Every form of exact knowledge is a knowledge of relationships. But if again you ask me what relation is, I find it difficult to tell, although nothing seems more familiar if you do not ask me. I will not attempt in the present discussion to answer all the philosophical questions that may be raised about the nature of a relation, but this I may at once say: a relation considered as the relation of anything you please to anything else, a relation such as that of a brother to a brother, of a debtor to a creditor, of an equal to an equal, of a king to a subject,— a relation, I say, is the character that a thing possesses when that thing belongs to a certain collection and where the thing loses it in so far as the thing can get out of that collection. Thus, a member of a club has a certain relation to the other club members. That relation belongs to him as long as he belongs to the club. If he leaves the club the relation lapses. The king has a certain relation to his subjects. It exists because the king and his subjects form a particular kind of collection. If the subjects rebel or vanish the king loses his character,— no longer has his relation,— ceases to be a king. The fact that objects can possess certain characters when they are in certain collections and lose these characters when
they leave the collections is a fact that for the time being we shall regard in this discussion as fundamental. So much then about relations in general. Now as relations are a very familiar fact so they are in a sense very various fact. It seems at first sight hopeless to try to classify them. Yet for our present purposes there is one classification of relations that it is necessary for me to make, because if we understand that classification we shall be able to understand better the nature of series, and so better the whole conception of order, and in the end we shall be able to understand better why orderly arrangement is so vital, both theoretical and practical. Relations might first be classified according to the size of the collections in which they hold. Thus obviously a king needs in general a great many subjects, and one who declared that he was a king, but who lived on a desert island with only a single subject, would not appear to us to be worthy of the name. Exactly how many subjects a king must have in order to be a king, we cannot say, but kingship is a relationship that involves a decidedly large collection. Instances of similar and more definite collections with attending relationships we see in the case of such collections as a baseball nine. The pitcher of the nine stands in a definite relation to every other member of the nine. His position as pitcher then determines a system of relations which belong to him in this collection, and which he would lose if he were no longer a member of the nine; but the relations themselves belong to him as a member of a certain group consisting of the nine itself and including perhaps the substitutes. The size of this collection is limited. Some relations are characteristic of collections of three objects. Thus, if I tell you that
on a bench there is one man who is between two other men, you observe that the property of being between is a property that belongs to one member of a collection of three. Finally, the most familiar instances of relationship are characters that an object possesses as a member of a pair of objects. Some people are fond of limiting the use of the term to this case. As a fact such limitation is unfair, yet on the other hand, pairs of objects present certain relationships which are extremely familiar and which especially interest us in ordinary life. Thus when I say that A is in front of B, I mention a character that A possesses as a member of a pair which pair consists of A and B. Similarly, when I speak of A as a debtor of B or as a lover of B, I mention each time a relation which exists between two objects, or rather a character which belongs to one of them as a member of this pair of objects. Now such relations existing within a pair of objects only sometimes get the technical name of dyadic relations, and as I said, dyadic relations are a most familiar one and most such relation names as friend, lover, father, servant, debtor, creditor, enemy, etc., are names for such relations. Let us hereupon fix our attention upon these dyadic relationships and observe how they may be classified. There is one classification which has great technical interest and also a very obvious practical interest. If two objects constitute a pair, they may be on even terms as members of this pair or they may be in some sense unequal members of this pair. Thus if you are speaking of the world of man, and you say that A is the brother of B, then at once in this pair B has the same character that A has; B is the brother of A, and that is the nature of the relation. It is what one may also call a reciprocal relation. It is
a relation such that if one of the members of the pair has it, the 
other also has it in the same sense. Again, if for any reason you 
say that A is the equal of B, you say that B is the equal of A. The 
relation of equality is reciprocal or mutual. But there are dyadic 
relations which are distinctly not mutual or reciprocal. Thus if 
A is the debtor of B, B is in so far of course the creditor of A, 
and the relations of debtor and creditor have a sharp contrast. If 
A is the superior of B, B is the inferior of A. If A is greater than 
B, B is less than A. Now relations of this sort, relations in respect 
of which the two members of the pair in question are not on the same 
level, are not equal, are called non-reciprocal relations. There is 
another and more technical way of naming the distinctions between 
these relations which we shall here find convenient. The relations 
of the first type which we have called the reciprocal relations are 
often and conveniently denominated symmetrical relations. Such I 
repeat are the relations of brother, friend, equal, companion. You 
must observe at once that these relations have a certain practical 
interesting character. They are relations of comity. They are the 
relations of the type which we sometimes have in mind when we speak 
of fair play, or of even justice, or of something of that kind. 
Nevertheless these relations are of very great importance as you shall 
see in regions of thought to which such practical considerations do not 
apply. The relations of the other type are called unsymmetrical. 
They are, I repeat, such relations as debtor and creditor, greater 
and less, higher and lower, father and son, etc.

So much for a very simple classification of dyadic relations which 
in a moment we shall find very important for the purpose of under-
standing the nature of series. I now am obliged to trouble you with
another classification of relations. A relation may hold within a pair of objects and there may be another pair of objects within which the same relation again holds, and now these two pairs of objects may have a member in common. Thus for instance A may be the friend of B, B may be the friend of C. Again A may be the equal of B, and B may be the equal of C. Or A may be the debtor of B and B may be the debtor of C. Whenever such a situation appears the relations in question may have one or another of two very markedly distinct characters. Consider for instance the relation of equality. As they say, things equal to the same thing are equal to each other. Hence, if A is the equal of B, and B is the equal of C, it follows inevitably that A is the equal of C. The relation of equality is then one which enables me to do what Professor James has called "skipping intermediaries." When I once know that A is the equal of B, and B is the equal of C, I can skip B altogether and say at once that A is the equal of C. Now there are a great many relations of this type. The relation of greater and less for instance is of this type. If A is greater than B, and B is greater than C, A, B, and C being for instance quantities, then A is greater than C. I can here skip the intermediary although I am no longer dealing with the relation of equality. If B is to the right of A, and C is to the right of B on the line, then C is to the right of A, and the relation of "to the right of" is again a relation which enables me to skip the intermediary. If B is in front of A, and C is in front of B, then C is somewhat remotely of course, but still really in front of A, and here again is a relation which enables me to skip an intermediary. Relations of this sort have a technical name; we call them transitive relations. They are relations which enable us to skip intermediaries
and to proceed to certain conclusions. You will observe at once that such relations can be themselves either symmetrical or unsymmetrical. Thus, equality is a symmetrical relation, but since it permits me to skip an intermediary and the reason that if A etc., equality is also a transitive relation. On the other hand, the relation of greater and less is unsymmetrical, but it is transitive, and in so far is like the relation of equality. But some relations, both symmetrical and unsymmetrical are not of the transitive type. Thus, if A is the father of B, and B is the father of C, the relation of fatherhood proves to be not transitive for then A is not the father but the grandfather of C. On the other hand, the relation of ancestor and descendant is transitive. If A is an ancestor of B, and B is an ancestor of C, then A is an ancestor of C.

Herewith we reach another classification of our relations. Some of them are transitive, and some of them are intransitive. This classification again has a very great practical importance. Upon transitive relations depends our whole power to arrange our world in linkages, as it were, so that by intermediate steps we can pass from one thing to another. Every chain or rope is an example of a sort of transitive relationships. If you pull the end you pull the middle, and thereby you pull the other end. If you move one link of a chain you move the next, and so on to the end of the chain. The locomotive is bound to the last car in the train by transitive relations, which, passing through intermediaries, make it so that after all the locomotive pulls the last car. Wherever many things are linked they are so by transitive relationships, or at any rate, by relationships which can be interpreted as transitive dyadic relationships. On the other hand, relations which are not transitive help to break up the
world into disconnected groups, to separate things, to keep them apart. Some relations are so to speak intermediate between the two types, being sometimes transitive and sometimes intransitive. The relation of friendship is an instance of this sort. If A is a friend of B, and B is a friend of C, that tends to establish a reasonable probability that A under favorable conditions should become the friend of C, and the relation there is a fact frequently proved transitive. On the other hand, as we know, the transition is by no means always made.

Well so much for our two classifications of relations; symmetrical and unsymmetrical on the one hand, transitive and intransitive on the other. But now in mentioning all these cases I have been intending to prepare the way for an understanding of the nature of series. How can I accomplish this? I said in beginning a series is a collection of objects. I now add, wherever there is a series there is a single transitive and unsymmetrical relation which links together the members of this series in such wise that if you choose any two of them, one of them stands to the other in this transitive unsymmetrical relation. Thus, a row of men constitutes a series. There is a transitive unsymmetrical relation. The relations of before or after. If you choose any two men of this row, one of them is before the other, and the fact that this same relation binds any pair in such wise that one of the pair is before the other enables you to arrange all the members of the series in a single row. Precisely so with the numbers. There is the relation usually of greater and less. It binds any two numbers together in such wise that one of them is greater than the other, and that fact makes the numbers stand in one series. Now I say that wherever we have a series we have such a transitive, unsymmetrical relation which binds any pair of things that
belong to that same series. This relation remains constant throughout the series. It is a relation upon which the series depends. But now let us turn our attention once more to the relations of the other type that I mentioned, the symmetrical relations. Symmetrical relations I said may be transitive, so the relation of equality is transitive. Now you will observe at once that if I have a large collection of objects of which I merely know that any two of those objects are equal to each other in some respect,—in size, in weight, or in capacity, or whatever you please,—then those objects are indeed set in no sort of order by this fact. Thus a collection of weights, all of the same size, do not so far constitute an order, whereas if I had a set of weights that could be arranged in a single series that would be because some such relation as greater and less held between any two of them. If I have then a collection of objects, any two of which are bound together by a single symmetrical transitive relation, such as equality, these objects do not of themselves constitute an order, but such a set of objects may form a very important part of an ordered system. Of course when I correlate or bring into connection two or more series of objects I do so by establishing symmetrical relations amongst the members of different series. Thus when you get a train to run on time, you establish certain symmetrical relations between the train and any station that you please on the road. Namely, the train and the station are together at that moment of time, but the relation of being together is a symmetrical relation. So correlation depends in general upon symmetrical relations. These relations may be transitive, and in that case they enable me to correlate a great number of series. Now when I look back upon any of those complex instances of order which I
discussed earlier in this lecture, you will see if you examine these cases carefully that in all of them the facts with which one deals are either arranged in series or else are in places which in various series correspond, so that they are in some wise correlated together.

And now let me define one more term. When a collection of objects is such as to be bound together by a single symmetrical transitive relation, I call this collection of objects a level. Thus the things that are in two pans of an apothecary’s scales are on a level if the scale pans balance. The places that are on a level with the sea are in a symmetrical transitive relation to one another. When in a large social order a great number of men are legally or socially equal one to another, these individuals constitute a level. And now if you look over the various orders that I have spoken of, you see that what we do when we deal with such orders in so far as we consider the correlation of various series is to arrange various objects on the same level. Thus while levels themselves do not directly put things into order they help us to bring various orders into correlation with one another.

We began our discussion with the inquiry, what do we mean by order? We have found that order consists in the existence and correlation of series. We have discovered that where various series are correlated we have what may be called levels. We have discovered that the arrangement of things in series has both a practical and theoretical importance. We have discovered that the correlation of series has also a practical and theoretical importance. We have seen that both the formation and correlation of series depend upon the establishment of certain relations which again have both a theoretical and practical importance. These relations are of two types;
the symmetrical transitive relation, such as equality, the unsymmetrical transitive relation, such as greater or less. We have seen that these relation types have their practical importance. What we shall have further to see at our next lecture is the way in which these relations and the orders which result from them help us to an appreciation of the laws of nature.
1.

The term law is used in a very great variety of senses and one sometimes feels that such usage has made it so vague that one would prefer not to employ it in designations that pretend to exactness. Still the word is once for all there, and it is supposed to be of great importance to us to become sure that nature is subject to law. We ask the question with which these discussions opened, namely what is the general lesson that the study of science has taught us and the most familiar answer is that we have learned from the study of science how nature is everywhere a realm of law. Since in many cases the laws of nature as discovered by the special scientists do not seem to have any very close relations to our own ideals and human interests, it is not infrequently asserted that when we study science we come to deal with a world that seems hard and cold and opposed to the aspirations of the heart. Yet precisely as we have found that the arrangement of objects in series and the grouping of other objects upon the same level with respect to a given series is a process that expresses human activities and that when accomplished involves the success of human efforts, so we have now to see that every time that a law of nature is discovered the practically interesting process of the human mind leads to the successful issue. Whatever ideals a man has to give up when he studies science, he certainly wins something in return for them. The view of the world that science gives us is everywhere a rational view, and in so far the ideal view. We are interested in finding law in our world of experience. What I wish to say to you on the present occasion is what interest the search for law in experience meets and what ideals we accomplish when we discover that law exists. To this end I must begin with somewhat vaguer and less definite conceptions of law and then quickly pass to those instances
which permit a sharper characterization of what it is that we look for when we seek for law, and what we find when we discover law. In the vaguest sense I know something that I can call a law whenever I can make with confidence a general assertion. Whenever I can say that all men are mortal or that all summer days are long, I may be said to be acquainted with a law, but the law that appears in this form is not one that stands for any very highly developed type of scientific knowledge. Even within the range of common-sense we are not content to know laws of this type and these alone. When you go shopping you are not content to be certain that everything that you buy must be paid for. You need to know the various prices of various things. In general, if you are considering what to purchase, you arrange various purchases and prices in series, or again upon a level. You consider that this object will cost more than that, or that any one of these objects may be bought for a given price. Only when you have a considerable number of assertions of this sort in your possession, only in other words when you have reduced the prices of possible purchases to order, have arranged facts in series and upon levels,—only then can you say that your shopping is conducted according to some kind of system, and that your knowledge of the present laws of the market is a knowledge that approaches exactness. Well now what holds in the world of common-sense holds also in the world of science. Propositions of the form, all A is B, and therefore if A occurs then B occurs, are frequent propositions in science, but no one such assertion is enough to constitute a scientific insight into the connections of facts. We know that all men are mortal, but such knowledge is not scientific. The knowledge that an insurance company possesses about the average percentage of the healthy men of a given age who will die within ten years is a knowledge that in its own way
approaches the scientific type. It is knowledge of what we call the statistical kind, and knowledge of this sort always relates to series of facts, and not merely to single classes of facts. The life tables show what proportion of a given class of men may be expected to die within a year, within two years, within three years, or within ten years. And the use of mortality tables depends entirely upon the construction of such series and of the various levels that go with them. Our knowledge of the laws that relate to the consumption of fuel is a very good instance of the sort of knowledge of law that really interests us in dealing with nature. To know that whenever you want to run an engine you must burn coal is not yet to know very significant laws of nature. The question that you desire to answer when you have to deal with a particular engine is the amount of fuel that you will use in a given time in case you are to get a given amount of work from that engine under the given conditions. Knowledge of this sort is inevitably related to a knowledge of facts that form a series. If you burn so many tons of coal you will get so much work done. A given increase in the amount of coal burned will produce a given increase in the amount of work to be done, and so on. Such knowledge becomes at once valuable for deciding how much coal must be put into a given ship in order that she may make a certain voyage. Again, how much coal must be purchased by the manufacturer in order that certain goods shall be turned out? You see then that it is almost useless to say that when we study the laws of nature we are endeavoring to find out merely general propositions. Our various general assertions must be assertions about objects that form a series, or again, about the relations of various series to one another. I may put the same considerations in another form. Textbooks of induc-
tive logic, that is, of the logic of science, very frequently tell you that the purpose of scientific investigation is the discovery of causes. Now a cause, such books say, in physical science, is an antecedent such that if this antecedent is present in a given case a certain consequence will follow. Examples of a knowledge of causes one finds in the textbooks in a case like this: An inquiry arises as to why a feather and a bullet fall with unequal velocities when they are dropped in the air, the bullet coming at once to the earth, the feather floating slowly down. The hypothesis arises that it is the resistance of the air which causes the feather to descend slowly. This hypothesis is tested by a well known experiment,—the experiment of permitting the bullet and feather to fall in an exhausted receiver. The air is here to a very great extent eliminated. The bullet and the feather fall in almost exactly the same time. And so one concludes that the air is indeed the cause of the ordinary difference in the behavior of the two bodies. Such, say the textbooks, is scientific knowledge, and unquestionably such knowledge is a fragment of scientific knowledge. I do not deny its importance as such a fragment. What I wish to point out is, however, that physical science does not merely consist of setting side by side a large number of essentially fragmentary assertions like this one. Physical science is interested in quite another aspect of such laws besides the aspect that can be expressed in a single general proposition.

John Stuart Mill in his logic laid considerable stress upon a method of investigating nature which he called the method of concomitant variations. You wish to find whether the air causes the various bodies to fall with different velocity. Accordingly you study various cases where the air has been partially taken out of a receiver, where the air has been still more completely taken out of a receiver, and so
on until you come to the case of a receiver as nearly exhausted as for
the purposes of your experiment it is possible or convenient to
exhaust it. In such a case according to Mill you reason that
whatever antecedent varies in such wise as to be attended by variations
of a given consequent must be a cause or at least part of the cause
of that consequent. Here one speaks as if the study of a great num-
ber of cases where a given supposed cause varies in amount, has as
its principal object the discovery of the one proposition that the
cause which thus varies must indeed be the cause of that effect which
is found always varying amongst the consequences present in the
various instances. Similar is the reasoning according to such
account when one notices that heat expands a large number of substances.
One finds for instance that the more one heats a bar of iron the longer
the bar grows, and that similar concomitant variations are observable
with a great many different kinds of substances. Mill's treatment of
the method of concomitant variations is usually interpreted by the
student as if the principal lesson of it were that in view of these
manifold parallelisms between the variation of a certain cause and the
variation of a certain effect, we can generalize and say this cause is
indeed the producer of this effect. I do not suppose that Mill
regarded such a generalization as the sole or as the principal signifi-
cance of his method of concomitant variations. What I wish to
point out here is, however, that as that method is very generally
interpreted by students the essential point of such investigations
is missed. What interests us in case we discover that certain
variations of A are attended by certain variations of B is not the
mere generalization which under such circumstances may become very
soon a common-place, namely the generalization that A is at least a part of the cause of B. What interests us is the correlation of two series of facts. The various stages, degrees, amounts of A, and the various resulting amounts of B. You may know some such generalization as the so-called law of the relation of demand and supply. You may know that if the beef supply in Chicago is hindered by strikes or by combination the price of beef in your market bill will rise. But the interest of the householder is not in this generalization. What he wants to know is how the price of beef varies. In other words, he wants to know such series of facts. If he becomes interested in tracing the causes of these facts, his interest will inevitably be to discover if possible how the variation of the supply of beef in Chicago is related to the price of beef in his market bill. He can discover this only in case he can correlate two series of facts.

And so now I say our knowledge of nature assumes the form of exact law precisely in so far as we know how various series are correlated together. So much for our first view of what we mean by knowing the laws of nature. It is because of this interest of ours in correlating various series of objects that the processes of measurement become of such great importance for scientific work. Exact science is possible, so we are often told, solely because of measurable facts. This assertion is logically speaking not universally true. But the extent to which it appears to be true is mainly due to the fact that whenever we are capable of measuring objects we are capable of arranging measurable objects in perfectly definite series. Assume for instance that I am able to weigh the amount of coal in a ship's bunkers or the amount consumed in a given time, it is possible to correlate the consumption of coal with the length of the voyage, with
the speed of the ship, or with some other such fact, this other fact itself being capable of measurement, so that different speeds of the ship, or different lengths of voyage may be exactly correlated with given amounts of coal burned. There is then nothing magical about the importance of quantity in science. The value of quantities in measurable objects generally is that they can easily be arranged in series and the consequent various series can be correlated together. But quantities are not the only objects that can be arranged in series. The points on a line constitute a very definable series of objects, although the points themselves are not measurable quantities, and although it is possible to consider their relative positions without measuring lengths. The fact of a highly exact science of a branch of geometry is possible, and exists, wherein no sort of measurement is used, and one is dealing merely with the relative positions of various points. This branch of geometry is precisely exact as that popular and more familiar branch of geometry which turns upon measurement. In what is called non-metrical geometry one correlates various series of points; declares for instance that the points a, b, c, and d on a line form a particular kind of row with b between a and c, and c between b and d. One further proceeds to prove that such so-called ordinal relationships amongst the various points remain unchanged when you pass from a given line to the shadow or projection of that line on a given plane. Such geometry continues by making assertions with regard to whether two lines have a point in common, and what point they have in common. A prodigiously exact science can thus exist in which nothing is measured, but everything is ordinally determined and the whole system of facts with which one is dealing constitutes an exact order with precise series and definite levels by means of which
definite series are correlated. I can conceive an absolutely exact moral science, remote enough from our present inadequate knowledge of our moral relationships, but such as some higher being might have. This exact science would determine precisely what act rightly or justly corresponds to a given situation. The laws of such acts would be laws that you would have to express in the form of complex series interwoven in systems. But neither acts nor their consequences, nor the situations in which they are required would need to be measured. Everything would have to be reduced to order. And everything would be reduced to order by means of establishing certain series. Thus, one would arrange possible morally significant situations in series. One would arrange corresponding acts in series. One would declare by a correlation of these series what act or acts properly corresponded to definable situations. And one would express all these correlations in terms of moral law; being in possession of such a complete moral system of the universe, would know precisely what to do in every possible situation. He would have an exact moral science, but he would not be measuring anything. The general relation of our conception of law to our conception of serial orders of facts having thus been suggested, I shall now try to proceed to a somewhat more detailed statement of what happens when we try to reduce our knowledge to scientific order with respect to the laws of nature.

In order to proceed further in this inquiry I have to introduce yet one more technical word. It is the word that you will have seen in my title, the term transformation. By transformation I mean the same as any definable change. A mere change may seem to us very unintelligible, in case it is sufficiently novel or violent or rapid. Thus when a flash of lightning comes it may be hard for us to tell
precisely how the flash seemed to be related to ourselves, what it
direction it came from, and so on. A sudden and violent physical
pain seems to us surprising, but unless the pain lasts some time,
at we find it hard to describe what point the pain began. A patient
at the outset of an illness whose nature is unknown to him finds it
difficult to give an account of his own symptoms to his physician.
If for the first time you feel a Leyden jar shock your impression is
that something of great importance has happened but you find it very
difficult to say what has happened, except in so far as you are sure
that you do not wish to repeat that experience. When the dentist
pulls a tooth a similarly unintelligible change of one's state of
consciousness occurs. It arouses tremendous repugnance, but resists
analysis. On the other hand by a transformation I mean a change in
so far as it is conceivable, definable, analyzable. If I pick up
this object and remove it from this place to that place, a change not
merely occurs but I observe what it is. A journey made according to
a plan constitutes a definable series of transformations. A series
of events constitutes a set of transformations precisely in so far as
the historian can tell us what happened. Now it is obvious that
all our knowledge of the laws of nature is a knowledge that has to
do with the transformations of nature. The world of our experience
is a realm where phenomena are constantly shifting, so that new things
get substituted for old. So far as our knowledge of these changes
remains vague, indescribable, indefinite, the changes may be real, but
we do not conceive them as definite transformations. The changes of
the weather, vaguely noted by our passing consciousness do not in
so far constitute scientifically definable transformations. But the
Weather Bureau, taking hourly observations, defines the transforma-
tions of weather that occur in the country during the day in a pre-
cise order. And of course all scientific knowledge of the weather
must be obtained by such exact analyses of transformations.

And now when we study nature, of necessity we are not interested
in single transformations, but in long series of transformations.
That at ten o'clock in the morning of a given day the temperature in
a given place rapidly fell--- this is a transformation which may for
a given purpose be regarded as a single transformation. A know-
ledge of the laws of the weather depends upon knowing long series of
such transformations. Here again serial knowledge inevitably pro-
ceeds and lies at the basis of all the study of the laws of things.
But what constitutes a single series of transformations? Anything
that happens in the universe is succeeded shortly afterwards by some-
thing which happens, it may be in a far distant place in the uni-
verse. Thus every event may concern entirely different objects or
people. Thus the eruption of Krakatoa on the earth may have been
followed some time afterwards by an event which occurred on the moon
or some distant star. It seems inconvenient to arrange such ap-
parently miscellaneous transformations in a single series. Even
historians whose reports of events have often to be very miscellaneous
are anxious to reduce their narrative to order by putting in a single
series of events which as they say belong together and so constitute a
single set of transformations. A chapter of the history of France
may be put in a general history by itself. It may then be followed
by a chapter relating to some nearly contemporaneous events in the
history of England. And so one may proceed from country to country.
However, when one's facts are arranged there is a strong interest in
regarding certain facts as belonging to one series and certain other
facts as belonging to another series of definable changes or transformations of the natural world or of human experience.

If we ask now what principle determines us to call a given set of events, events in some one series, and to regard certain other events as belonging a different series, the answer is not difficult to give. A single set or series of transformations always means for us a set or series of transformations which appears to us to leave something constant, something changeless throughout the series. We estimate transformations by contrast with permanence. In a world when anything was followed by anything, and where not the least constancy could be observed, we should have no reason to arrange transformations in serial order, or to conceive them as constituting connected systems of events. The series of changes which take place when the steamer during the voyage consumes her coal is a single series in so far as despite the changes in the coal something remains constant throughout the voyage, namely, the ship, her destination, and whatever else it is in terms of which we recognize the series of incidents in question. A series of events in the history of France is characterized by the fact that even if this series occurred during the French Revolution when tremendous changes were in question, something, namely, the French people, remained in some notable respects the same, throughout that series of changes. A historical narrative whose scene and people change with every new event mentioned would have no sort of unity. In short, the unity of a series of events is definite in what remains permanent to that series of events. So far we have in our possession these results; that our knowledge of the laws of nature is a knowledge which has to do with series of events; that a series of events is defined as a single series by
virtue of something that remains constant throughout that series; while our knowledge of the laws in question depends upon the correlation of various such series. Having premised these considerations, I may now proceed to assert in general that when one knows a law of nature one is aware that a certain set of changes or transformations forming a series is such as to leave unchanged or unaltered certain definable conditions. If the law of nature that one knows is a complex and important law, one's knowledge goes further. It then consists in ascertaining that a great many different series of events are such that the type of transformation present in all of them leaves unchanged a particular type or character present in the things which you are considering. The statement now made becomes by itself too abstract, and must at once be illustrated. When the officials in charge of navy supplies know such laws regarding the consumption of coal as enable them to determine what supply each ship should have for a given purpose, the laws that they know relate to some such complicated relations as the following: If any ship sails she will burn coal. The series of events consisting of the tons of coal burned by a given vessel on a given day, will determine the time which will elapse before all the coal in the bunkers is consumed. Now the officials who determine the supplies of the navy will be able to predict for a given ship within limits how much coal she will burn during a given voyage. By virtue of what knowledge of law are they able to make this prediction? By virtue I say of their knowledge that a great many different series of events, each one of which consists in the burning of coal by some one ship. Now a series that have certain characters in common; that is, if you pass from one such series to another, you find that the coal burned has a certain con-
stant relation to the work done. This constant relation remains
unchanged no matter how many different ships and how many different
voyages you are considering. Or if it does not remain actually
unchanged, that is precisely because the laws of the process in ques-
tion are not fully known and the influences which determine the
consumption of coal are so complicated, including the habits of
engineers and coal heavers, that you are unable to say precisely
what will be always the amount in case of all the voyages and all the
ships. Supposing the loss to be known, the laws always take the same
monotonous form. Consider a great many series of events. In each
one of these series something remains constant. If you pass from one
series to another this constant will itself change, as the ship that is
engaged in one voyage is different from the ship engaged in another,
and the coal used is different. But still as you pass from series to
series something will still remain unchanged. If you correlate such
series as the ships burning their coal with such series as stationary
engines on land burning their coal, and finally with such series as the
student of the theory of heat considers in general, you get still
wider and wider generalizations. They all have the type that great
numbers of series are in such wise correlated that this or that re-
mains unchanged as you pass from series to series. Since your passing
from series to series constitutes itself a more or less ideal set of
transformations, you reach the final result that all our knowledge of
law assumes this form, that certain series of changes, real or ideal,
constitute such a system that a definable something remains unchanged
throughout these series. The ideas of which I am dealing are here
decidedly technical, but the illustrations are simply countless. Let
us exemplify the whole process in still another way. A socially
skillful person knows how to behave in various company. This knowledge he has with regard to the proper behavior is of course founded upon a knowledge of the laws of such social groups as he is accustomed to associate with. Now any occasion when a man meets a company of people presents to him a set of transformations. The people speak, move about, co-operate, come and go. These transformations on any given social occasion are such as to leave unchanged certain conditions, namely the conditions upon which civilized society in general, and this social occasion in particular, may depend. The socially skillful person now not only follows any such series of events with success, but when he passes from social occasion to social occasion, when he meets now this company of people, and now that, he knows how to behave and to succeed on each new occasion. But in so far as he is aware of his skill, in so far as he knows upon what his success depends, his knowledge consists in an acquaintance with certain principles of conduct which remain unchanged whatever the company or whatever the social occasion. Such a knowledge of the laws of social occasions is of course very insignificant. It is practically liable to all sorts of error. With most socially skillful persons a great deal of such knowledge remains unconscious. Every social life consists in so much of definable transformations, as of more or less fascinating, surprising, or appalling changes in the social environment. But so far as such knowledge is exact, is conscious, is such as to lead to success, it is always a knowledge of what relations, character, principles, modes of behavior,—the transformations of the social order always leave unchanged. The socially successful man who comes into a new social group has in so far very greatly transformed his social situation. He no longer is where he was or amongst such
people as he has been meeting in the past. So far as he still adjusts his conduct to the situation and knows why he does so, he keeps on following such principles as he has already established. In other words, the very novelty of his conduct, the skill with which he adjusts himself to a new situation, depends upon preserving intact exactly those principles of conduct by which he succeeds. He acts well this time upon this novel occasion because he has rules that he can hold constant, while these rules are precisely such as determine what he has to do upon this occasion. So here again the knowledge of law is the knowledge of what remains unchanged amid transformations.

I have thus defined the nature of the knowledge of law by first speaking of the series of facts whose laws we discover and by then pointing out that our knowledge of law is the knowledge of what some series of changes inevitably leaves unchanged. I could approach the whole subject in another way and repeating considerations which occurred to us at the outset I could illustrate how no knowledge of what is changeless in the world is of the least service to us unless it at once is related in our mind to the definition of a series of objects, and in fact to the definition of a series of changes. What is it that always happens in the world? I can of course answer life always goes on, the sun always shines, men die daily, in brief, I can utter any common-place that you please about life and the world. In so far I do not name any notable law which is of real scientific importance. But as soon as I say this time when you are preparing coal for this voyage you need so and so much coal, because this case has a definite place in a series of possible cases, a series consisting of different possible ships with different possible voyages and different possible amounts of coal needed,--- then and then only do I make use of a definite knowledge of the law. But my knowledge of law
now takes the form of saying that since all the members of this series are of a certain type, this particular member occupying this place in the series must be of a particular character, just in order that it may conform to the general type. In the series of whole numbers, each new whole number is a single case. But of course the whole series is formed according to a certain law, and all the whole numbers conform to the type of this law. This whole number in its own place in the series must have some special character in order that it may here illustrate the general law. The general law of the whole numbers is that each one follows a particular number and precedes a particular number. In order that seven for instance should obey this law, it must follow six. And as the follower of six it has to have special character which results from its place in the series. Hence, series and laws are inevitably correlated. If I begin with a series, it becomes of scientific importance to me in so far as I observe some law, it either holds in a single series or holds when I pass from series to series. On the other hand, so far as I define law I define what is a mere common place unless I apply it to the series of fact and so determine what this fact must be in its place in the series, because of a general law that is verified by all the facts that form the series.

So much for a sketch of the way in which laws and series go together in our knowledge of nature. But now for one or two illustrations of how the notion which we defined at the last time, the notion of a level, is related to our conception of a law of nature. When I set a great number of series side by side in my mind, and so correlate them that certain facts belonging to these various series occupy a corresponding place in all the series, then as I said at the
last time the facts in question lie on what one may call a level. Thus, if I correlate the series of facts that I meet with ascending one mountain with the series of facts that I meet with in ascending another mountain, one very convenient way of carrying out this process is furnished me by the familiar process of defining how high above sea level any given point on any given mountain is. Now all the places on the earth's surface that agree in being 1000 feet above sea level, are themselves upon a certain level. This level may determine certain similar constant characteristics which I can verify from place to place on the earth's surface. Thus for instance within limits the air pressure at a given level tends to have the same relation to the air pressure at sea level at the same moment whatever part of the world I may happen to be in. In this way the correlation of the various elevations above sea level is carried out by what one may call a series of levels. Since wherever we correlate various series the same considerations come into sight, we may observe that most, if not all, instances where we speak of laws of nature, can be expressed by saying that various facts in various real or ideal series stand upon the same level. For as we have thus seen, to know a law of nature is to know something that remains constant when you pass from series to series. If various mountains, or rather elevated places, agree in certain respects with regard to the air pressure present at those levels, then I am in possession of a certain law relating to the connection between elevation and atmospheric pressure. But just as the sea level enables me to define this law, so the facts about the pressure at various places on the earth's surface might themselves be used to establish something equivalent to a level amongst all these places. If a great number of places on the earth's surface
agree at a given time in having the same pressure of atmosphere, these various places constitute a system of facts which stand in symmetrical transitive relations in so far as the pressure at any one place is equal to the pressure at another place. As soon as I ask what is the cause of this equality of pressure, and find its cause in elevation or in any other constant physical condition, I then inevitably set the places which have the same atmospheric pressure on a level, while at the same time I place in a series various places which have different atmospheric pressure. Consequently, in my mind I arrange all the places on the earth's surface with reference to atmospheric pressure in what may be called a series of levels. All those places which have the same atmospheric pressure being at a given moment on the same level, different places which have different atmospheric pressures being upon different levels, and all these levels themselves forming series.

To change the instance once more, if we return to the socially successful man in the various societies, he may define his laws to himself by defining certain classes of social situations, which in a certain respect are all alike, or are on the same level. Thus there is something that one does in greeting, there is something that one does in making or in returning compliments, in accepting or declining invitations, etc. All greetings, compliments, acceptances, acts of declining, and so on, are social facts, each state of each are on the same level. And now if one can define one's social laws one defines how one's acts would alter their character as the social levels in question change. Only in case one could define the possibilities of changing situations, the ways in which different types of invitations ought to be accepted or rejected, only in so far as one
could do this would one have significant laws of social behavior, and
as soon as one did, these various social levels would be again ar-
 ranged in a series.

In view of these considerations we may get a general view of
what happens in all our scientific thinking if we think of the
matter in this way. Suppose all the changes that occur in the world
to be arranged in a system, so that all the changes were formed into
series in so far as any set of changes gave good reason for regarding
them as members of a single series, while all these series were
grouped in some way side by side as lines may be grouped in a
diagram. Suppose then various cross lines such as levels are drawn
from part to part of this diagram, and suppose hereby one correlates
various parts of one series with various parts of another, then the
result is that one gets the sort of insight into the laws of things
that we human beings are constantly endeavoring to get.

My result so far is that since our knowledge of law depends upon
this endless arranging, grouping, and regrouping of series and systems
of series or transformations, our whole knowledge of the laws of
nature consists in such serial arrangement, and such observation of
what remains unchanged through varying conditions, and in consequence
includes all sorts of artificial rearrangements of our experience.
Our science is not a mere report of facts of experience. It is a
rearrangement of these facts. It is not an account of how the world
would look if one took no interest in these series that we so carefully
consider and arrange. It is an account of the world considered with
reference to such series as we find it interesting to construct. If
the facts decline to permit any such construction we should indeed
be helpless. That the world permits us to make such constructions is
if you like, something of great value in estimating what kind of world this really is. But on the other hand, our own constructive activity constitutes by far the larger portion of our scientific insight into the laws of things. When we arrange events in series we are never merely at the mercy of the order in which the events happen to occur in time. Even the historian artificially rearranges his facts, establishing those levels and distinguishing those series which interest him. In any series there must be something that remains unchanged throughout the series, but this something must be something that so appeals to us, to our human interest, to our practical concern in the fact that our attention is attracted by it. It is essentially we who remain unchanged in a given case when we observe a given system of facts. Our interest, our experience, our attitude, our valuation of certain facts, remain in such wise unchanged that we are able to appreciate these facts as forming a single series. Leave our interest our way of experiencing, our attitude, our fashion of construction out of account, and you have no sense left in which the facts can be said to form a series. On the other hand, when we correlate many series together, we do so in a distinctly artificial fashion. The levels that interest us are again determined by certain unchanging features which we find in various series as we pass from one to another. These unchanging features exist with such reference to our own interest, to our own attitude, to our own attentive deeds that if you abstract from our part in the process it is quite impossible to say what the facts alone by themselves would determine. The kind of significance which all this arrangement of facts in series, and of this study of the various levels has, our social instance has plainly shown us. The man to whom social situations form series exemplifying various
laws, because they are capable of being considered with reference to certain constant social interests and fashions of behavior, deals with the social world in a practical way and arranges his series and levels for the sake of success. The official coangling the squadron has to deal with facts, which indeed in one sense are stubborn enough, while in another sense they fall into these series and exemplify these laws because he approaches them with a certain human interest and because he wishes to accomplish a certain task. Abstract from the kind of success that we are seeking in our dealings with nature and you have no aspect of nature left which defines any precise system of laws. Beings interested in arranging facts otherwise, beings whose attention was attracted by other changeless aspects of their experience than those which interest us, might come to a different view of the laws of nature, as beings with wholly different sense organs from ours would reach. It is true that when we deal with nature we are not in the least privileged to be capricious. But on the other hand, we are not in the least able to abstract from our own forms of thoughtful activity and to define what the natural world would be if we had a wholly different type of conception.

We have now reviewed some of the fundamental conceptions of science, and we have found that they are distinctly human in their character as they are empirical in their illustrations. Nature permits you to conceive her fact in certain systems, but what these systems are your own interest as a thinker determines. Nature does not say to you, "Such I am and such I would be if you were not conceiving me." Nature says "Thus I present myself to your sense and thus I permit you if you are laborious enough and patient enough to succeed in rearranging my facts so that your industrial art makes its
triumphs and so that your scientific theories assume their well known
types." The vastest generalizations that we make concerning nature
conform to this principle as well as do the simplest. The great
principles of the theory of energy of which our modern scientific
inquirers rightly make so much are simply principles relating to
series and to levels,—the series and the levels in question being
such as have no meaning apart from certain very human processes of
description, definition, and rearrangement of facts. It is possible,
experience permits it, that we should correlate certain physical
facts with quantities of energy, with a series of facts known as
our own numbers, so that as we say we measure anything, and assign a
certain number to a certain energy, as naming the quantity of the en-
ergy present under the given conditions. All the objects that we
deal with in such a case are not simple experiences, but highly
transformed explanations of things which experience permits us to
define. After we have once thus arranged in series quantities of
energy, we are able to define a certain law or pair of laws which may
be said to govern the transformations of energy into physical systems
known to us. The first of this pair of laws is the so-called
principle of the conservation of energy. It is the principle that all
the transformations of energy of a closed system leave the quantity
of energy on the same level so that with respect to quantity of energy
all the transformations of a closed system constitute a level. The
other principle is that energy tends to pass from higher to lower
levels of intensity, within the same system. From this point of view
the series of transformations of energy, is a series whose states are
not upon the same level, but run through a certain irreversible series
of changes, energy in this sense passing from higher to lower levels
of intensity. The whole theory of energy is thus a theory of relating
to series and levels conceived energy. The human interests which
led to these conceptions are perfectly obvious to anyone who will
consider the conceptions concerned. When we say that the facts of
the world are of this type we mean the facts as conceivable and as
capable of arrangement by us. The conception and the arrangement in
question are very highly artificial. The arrangements in question
are made by us for purposes which are not only theoretical but prac-
tical. By making such arrangements we guide our conduct. We
understand nature in this field, precisely in so far as we can guide
our industrial conduct, and our conduct as scientific conceivers of phe-
nomena with actual success. When nature permits us to raise potatoes
I do not suppose that it is a very notable revelation of the absolute
nature of things, as they would be apart from human ingenuity, to say
that the world permits potato raising. When we say that the prin-
ciple of the conservation of energy is the truth, we substantially mean
that man makes a success, i.e., industrial and theoretical, on the basis
of arranging his conceptions of physical phenomena in a series and
upon the levels which the theory of energy employs. We have no
right to say that this fact reveals to us what the world would be
by itself if man were not there conceiving it. The success of our arts
and the success of our sciences is of substantially the same philo-
sophical significance. Nature permits both if we work hard enough.
Neither success is by itself sufficient to decide for us what the
ultimate nature of things may be.