My dear Royce:

If you have really detected a fallacy in my proof that two collections cannot each be greater than the other, it naturally concerns me much to understand it. Couldn't you jot it down. For you convenience I will restate my argument.

1. A relation being a mere logically possibility. To assert that an indesignate relation of only general description *exists* and to assert that it is *possible* are the same. If sets of individuals such as a relation of the described form requires exist, there must be such a relation. Thus, there is no one-to-one relation of every individual of the collection, *X*, *Y*, *Z* to an individual of the collection *T*, *J*, because if *v* be the [2] relation, and *x* be that one of the collection *T*, *J* to which *X* is *r* and *y* be that one of the same collection to which *Y* is *v*, *x*, and *y* must be different, and any unit of the same collection to which *Z* should be *v* would necessarily be either *x* or *y*, contrary to the assumption that *v* is a one to one relation.

2. Hence, if there be no one-to-one relation in which every *B* stands to an *A*, it must be logically impossible that there should be such a relation.

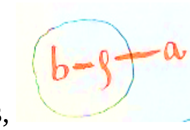
3. But a one-to-one relation is not in itself absurd, nor can there be any contradiction in supposing that there is a one-to-one relation in which every *B* stands to *something*, since in fact every *B* does stand in the one-to-one relation of identity to something.

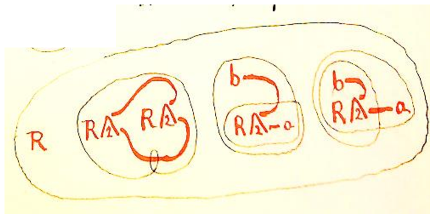
4. Hence the impossibility must consist in some existential limitation of the *A*s. To show what I mean by an "existential [3] limitation," take any other form of relation as an example. Suppose the relative s to be such that, whatever individual *X* and *I* may be, if *X* is s to *I* then there is *just* one individual other than *I* to which *X* is s and there are *just* two individuals other than *X* or each other that are s to *I*. Then if the *B*s, *by themselves*, are all s to anything, the multitude of *B*s is a multiple of 3. If, this being the case, nevertheless the *B*s by themselves, are in *no such* all to *As*, it can only be that *A*s enough do not exist. Hence, the only way in which to sums could be each greater than the other would be by both being sufficiently *small*; and in fact if both are *O* they are each other greater than the other, if you choose so to define.

5. However, in order to ascertain more clearly the nature of the existential limitation of the As.

Begin by supposing s to be such that  and also such that  That is to say [4] if *X* is s to *I* and *X* is s to *J* then *I* and *J* are identical, and also Every *B* is s to an *A*.

There are, if [b] is the multitude of the *B*s, $a^{[b]}$ such relations, that is there is a multitude of such relations greater than the multitude of *B*s. (Supposing there are more than one *A*s.)

6. But now suppose that in every such relation some *A* is not $_S$‘d by any *B*; that is, 

or more fully 

Now perhaps you think that at the next step I reason in this way. There is room in [5] the *coupé* for Mrs. Royce and there is room for Mr. Royce and there is room for Peirce & therefore there must be room for all those. But my reasoning is really this: There is room *under every combination of possibilities* for Mrs. Royce, there is room under every combination of possibilities for Mr. Royce & there is room under every etc. for Peirce; & *therefore* there must be room for all these. Perhaps you think that in some concealed fashion I am reasoning that because, no matter how many have got up there is always room for one more, therefore there is room for all.

But I submit that neither of those fallacies can be made out in my reasoning. It is a question perfectly analogous to one of room; — you might say it *is* a question of room. I am not [6] *assuming* that the correlates take their places in linear order, although undoubtedly it immediately *follows from my conclusion* that all possibilities might be reached in that way.

What then is my reasoning here? It is that there are $a^{[b]}$ ways in which every *B* is $_S$ to an *A*; — that is, there are that many different $_S$s. Bow if there is not one of these in which every *A* is $_S$‘d by a *B*, it follows that *under no possible combination of circumstances*, all of which are included among the $a^{[b]}$ $_S$s, is there any case in which the correlate of any *B* is logically necessitated to be a non-*A*, and therefore there must be some combination in which no correlate of a *B* is a non-*A*. [7] Let the correlates of $_S$ be changed so as to convert it, in every possible way, into a one-to-one relation. Then the question is whether every one of the vast collection of relations so obtained will have one or more non-*A*s as correlates of the *B*s. If this be the case, there must be a logical necessity that it should be so. But since it is expressly assumed that in all possible cases some *A* will be an un-$_S$’d by any *B*, and since the only way in which any given *B* will be compelled to take a non-*A* for its correlate (that is to say, in which there will be no possible variation in which that *B* takes an *A* for its correlate) is that there is no *A* that is not a correlate of some other *B*, it follows that under no possible [8] combination of circumstances is any single *B* prevented from having an *A* for its correlate. Now if no single correlate is or ever could be under any possible combination of possibilities compelled to be a non-*A*, there must be some combination of possibilities under which all *B*s should have *A*s for their correlates.

Can you put your finger on any fallacy there?

But going back to the point *P*, a second way of proceeding would be as follows:

If among the relations $_{s}$ hitherto obtained there is any which is a one-to-one relation, that at once proves my point. But if there is no one-to-one relation, change the correlates of each $_{s}$ in a manner to [9] be described, so as to render the relation a one-to-one relation. First, for the sake of formal completeness, I will say that, if possible, the correlates are all to be changed to *A*s that are not correlates. But it will never be possible to convert the $_{s}$ into a one-to-one relation in this way. For were this possible, the resulting one-to-one relation would have been one of the $_{s}$s. Secondly, (another formal division,) we will suppose that correlates are to be changed from *A*s that are correlates to *A*s that are not correlates so far as possible. What can prevent this being done so that no *A*s remain that are not correlates? It is purely an affair of necessary logic. It must involve some contradiction. But the only possible ways in which it could involve con- [10] tradictions, 1st, that some $_{s}$ should have no *A*s that were not correlates, and 2nd, that there should be no *A*s which could cease to be correlates of some *B*s without leaving them non-correlates of all *B*s. In the former case there would be a one-to-one relation in which every *A* stood to a *B*. In the latter case there would be a one-to-one relation in which every *B* stood to some *A*. What then? Shall we say that no change of correlates can reduce any one of the $_{s}$'s to a one to one relation? This seems manifestly absurd. But if the change can be made it can be divided into two stages, the first of which shall change correlates into other *A*s in all ways that are not self-contradictory, the second part changing correlates into non-*A*s. But this second part will never be reached, because no contradiction ever [11] arise until our point has been proved in one or other of the two ways just mentioned.

To me, as at present advised, this seems an absolutely necessary argument. There is no assumption that the conversion is to be performed in indefinitely many steps; but only that *two* steps are to be taken.

However, going back to point, *P*, there seems to be a third argument equally conclusive and even more direct. Almost anybody but you would it convenient that I should repeat that the situation is that we have all the $_{s}$'s which result from first supposing all the *B*s to be in a relation $_{s}$ to one *A* and taking all [12] the variations resulting from changing the correlate of each *B* to some other *A*; and it is supposed that, for all these $_{s}$'s, some *A* is not $_{s}$'d by any *B*. That is, in view of the existential constitutions of the two collections, there would be a contradiction in supposing that every *A* was $_{s}$'d by a *B*. But the only way in which there could be a contradiction in supposing that some of the variations had produced such cases is by this no matter what $_{s}$ you take, in changing this $_{s}$, so as to make some (or all) of the *A*s that it leaves not $_{s}$'d, to be $_{s}$'d, every such change of it would cause some *As* that [13] had been $_{s}$'d to be left not $_{s}$'d. [Of course, in general, even if all the *A*s that has been $_{s}$'d were left wholly un-$_{s}$'d, still the *A*s that had not been $_{s}$'d would not all become $_{s}$'d; but I only mention this to point out that I have said nothing that conflicts with it.] But this being thus, there can be no contradiction in supposing that some of the $_{s}$'s so produced will be one-to-one relations; and therefore some of them certainly will be so. Consequently, there will be a one to one relation in which every *B* [14] stands to an *A*.

Now I am anxious to know what fallacies can be found in these three arguments.

very faithfully,

C Peirce

Transcribed by Joe Dillabough