1905 Aug 19

My dear Professor Royce:

I receive your important Memoir today & although I have not yet had time to read far into Chapter II, I will venture on a few remarks which may for aught I know be contained in the Memoir itself.

Beginning with the formal definition of an *O*-collection in §19, I will consider law I in the form

$$E\ (\alpha\gamma) \propto E\ (\alpha)$$

that is, *E* expresses such a relationship between the "elements" of any "collection" as necessarily remains when some of these elements have been suppressed. Further, since this relationship is one of equiparance, the assertion *E* (α) is that each "element" of α after the first pair is suited to preserve the *E*-relationship. It therefore expresses (does *E*) an *agreement* among all the "elements." If I ask what sort of agreement, the answer must be an agreement in any one of certain respects, which I may for convenience refer to as essential [2] respects. For if $\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4 \ldots\ \alpha_n$ all agree in any one respect of a given class, so necessarily do any of these α's fewer than *n*. Consequently *O* (α) expresses that in any "essential respect," the elements α_1, α_2, α_3, etc. are not all alike. Now let me see what limitation Law II places upon this interpretation. That law is that if *E* (δ) then either *E* (β) or else, for some element of β b_n we have $E\ (\delta b_n)$. That is, if d_1, d_2, d_3, … d_n (the "elements") of δ all agree in an "essential respect," and if b_1, b_2, b_3, … b_n do *not* all agree in any one "essential respect," then there must be some one of the *b*s, say b_n such that it does not agree with d_1, d_2, d_3, … d_n. This will necessarily be the case if and only if the "essential respects" remain the same for all "collections." For, this being the case, the *b*s do not agree with one another in that essential respect in which the *d*'s agree. Suppose for instance we call the character of *d*, in that [3] respect 0, so that $d_1 \propto 0$, $d_2 \propto 0$, $d_3 \propto 0$, etc then some *b* is 0 and some is $\bar{0}$, or as I may write it, is 1. Thus Law II adds nothing to Law I except that the essential respects are the same universe of respects for all "collections," *and there is merely a dual variation in each respect*. Let us form a table, or block, for each collection, assigning separate horizontal line to each "essential respect," and a separate column to each "element." I will denote the determination of the first element (the one arbitrarily taken as first) in each respect by 0 and the same character 0 shall denote agreement with that first in that respect, while 1 shall denote the opposite determination. Then identifying respects in which no two elements differ, the two possible "collections" of the system will be represented by blocks [4] of numbers expressed in the secundal notation

00	000	0000
01	001	0001
	010	0010
	011	0011

0100
0101
0110
0111

Thus *O* (α) means that the block of numbers representing α does not contain *zero*. Law II might be expressed: "Two Elements which disagree with the same element agree with each other, in any one essential respect."

Of the special principles, III, IV, V express that, there are at least 3 different (i.e. nonequivalent) elements in the system.

If each respect counted as essential there are elements that differ. (So that the whole system is an *O* collection.)

VI says: If there be an element *w* which differs in every essential respect from some element of a collection *l*, then there is an element of the system *u* which agrees with *w* in every respect in which all elements of *l* agree with one another and differ from *w* in all respects in which elements of *l* differ from one another.

§21. Let the determination of *x* in any respect be 0. Then since *O* (*xy*), the determination of *y* will be 1. Then since *O* (α*x*) and *O* (α*y*) one element of α will be 0 and another 1. Then in any respect the elements of α will be unlike or *O* (α).

§22. Since *O* (π), in every respect some two elements of π will differ. Suppose then that in a certain respect P_0 = 0 P_1 = 1. Then the obverse collection will contain γ_0 = 1 γ_1 = 0 and thus in every respect elements of δ will differ or *O* (δ).

§23. Let the determination of *x* in any respect be 0. Then by *O* (*xy*), the det. of *y* will be 1 in any respect. Since *O* (β*x*), the det. of *some* element of β will be 1, and since *O* (*lx*), the det. of *some* element of *l* will be 1. Since η and *l* are obverses, the det. of some element of η will be 0. Since in *any* respect the det. of some elements of β is thus shown to be 1, while the det. of some element of η is 0, we have *O* (β*y*). This is 93 words, Proof in Memoir 84 words. [6]

§24. Since ε consists solely of elements complementary to λ, every element of ε is in every respect different from some element of λ. Since *O* (δε), in no respect is every element of ε like every element of δ. SInce δ and γ are obverses, in no respect is every element of ε different from every element of γ. Therefore in no respect is every element of λ like every element of γ, or *O* (γλ).

There is one feature of your work which puzzles me; and it seems to me that this is because the explanations of the introduction are not sufficient.

The elements are "simple and homogenous." Then I do not see how two can be equivalent, or how *O* (*p*, *q*). I should think implied that two elements made up the system.

In §7 I learn that a collection "is determined wholly by the fact that certain elements do, while certain elements do not, belong to it." I should think, then, that (*x*, *r*) = (*x*, *x*, *r*). Is that so? [7]

Are your elements definite individuals, or indefinite individuals, or are they generals? This is a vital question, the logic of the three being different. If they are definite, I do not see how one can enter a collection twice over. If they are indefinite, they are signs, and equivocal signs. May the same letter denote two different elements?

I should like a clear explanation of these matters and an accurate statement of the sense in which an element can enter repeatedly into a collection.

Is is that besides elements and respects, there are also modes of composition, so that (*a*, *b*) and (*a a*, *b*) and (*a*, *b b*) and (*a a*, *b b*) etc. are different compounds, though of the same composition? This would seem to be explicitly negative [8] by §7.

very faithfully

C.S. Peirce

Of course two different equivalent elements differ in *inessential* respects.

Transcribed by Joe Dillabough